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Fluid flow and heat transfer through a porous medium channel with permeable walls

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INTRODUCTION

Transport processes through porous media play important roles in diverse applications such as transpiration cooling and design of solid-matrix heat exchangers. In recent years non-Darcy effects in porous media have received great attention. For instance, laminar steady fluid flow and heat transfer in a porous medium channel bounded by two impermeable parallel plates was studied by Kaviany [1]. With negligible inertia effects, it was observed that heat transfer is enhanced for porous media with large shape parameters. Recently, Khodadadi and Kroll [2] have reported analytic expressions for the fully-developed flow field with simultaneous suction and injection on both walls of the porous medium channel. In extending these contributions, a theoretical study of the fluid flow and heat transfer through a porous medium channel bounded by permeable parallel walls with equal suction or equal injection (Fig. 1) is presented.

TRANSPORT EQUATIONS AND THEIR SOLUTION

Following Kaviany [1], the momentum and energy transport equations are

$$\rho_f(\langle\vec{u}\rangle\cdot\nabla\langle\vec{u}\rangle) = -\nabla\langle P\rangle+\mu_f\nabla^2\langle\vec{u}\rangle - \frac{\varepsilon\mu_f}{K}\langle\vec{u}\rangle - \frac{F\varepsilon^2\rho_f}{\sqrt{K}}(\langle\vec{u}\rangle\cdot\langle\vec{u}\rangle)\mathscr{L} \quad (1)$$

$$\langle\vec{u}\rangle\cdot\nabla\langle T\rangle = \alpha_e\nabla^2\langle T\rangle \quad (2)$$

where $\langle\vec{u}\rangle$ and $\mathscr{L}$ are the volume-averaged fluid velocity vector and the pore velocity unit vector, respectively. In what follows, the symbol for the volume-averaging operation has been abandoned and the inertia term in the momentum equation is ignored. Following White [3], far downstream of the entrance the stream function (Ψ) can be defined as:

$$\Psi(x, y) = Hu_m(x)f(y^*) = (Hu_m(0)-Vx)f(y^*) \quad (3)$$

where $u_m(0)$ and $u_m(x)$ are the mean axial velocities at $x = 0$ and x, respectively. The positive and negative values of permeation velocity at the walls (V) correspond to equal suction and equal injection, respectively. In equation (3), $y^* = y/H$ and f is a non-dimensional function to be determined. The two velocity components are $u = u_m(x)f'(y^*)$ and $v = Vf(y^*)$. When these relations are inserted into equation (1), and the pressure terms are eliminated by cross-differentiation, the following fourth-order nonlinear ordinary differential equation is obtained

$$f'''' + B(f'f''-ff''')-\frac{\gamma^2}{4}f'' = 0 \quad (4)$$

where $B = VH/\nu_f$ is the blowing Reynolds number and $\gamma = 2H/(K/\varepsilon)^{0.5}$ is the porous medium shape parameter. For the case of non-porous media ($\gamma = 0$), the numerical solution of equation (4) and discussion of the pertinent results are presented by White [3]. The appropriate boundary conditions are the symmetry condition at the mid-plane and the no-slip condition plus the specified velocity values at the wall, i.e. $f(0) = 0$, $f''(0) = 0$, $f(1) = 1$ and $f'(1) = 0$.

Under the condition of thermally fully-developed flow with constant heat flux rate per unit channel length, constant heat transfer coefficient, and ignoring the axial conduction term in the energy equation, a second-order ordinary differential equation is obtained

$$\theta''-Pe\,f\theta'+(Nu-Pe)f' = 0 \quad (5)$$

where $\theta = (T_w-T)/(T_w-T_m)$; T_m and T_w are the mean and wall temperatures, respectively. Pe is the Peclet number ($Pe = B\,Pr$) and Nu is the Nusselt number ($Nu = Hh/k_e$). The appropriate boundary conditions are: $\theta'(0) = 0$ and $\theta(1) = 0$. In addition, one can show that: $Nu = -\theta'(1)$.

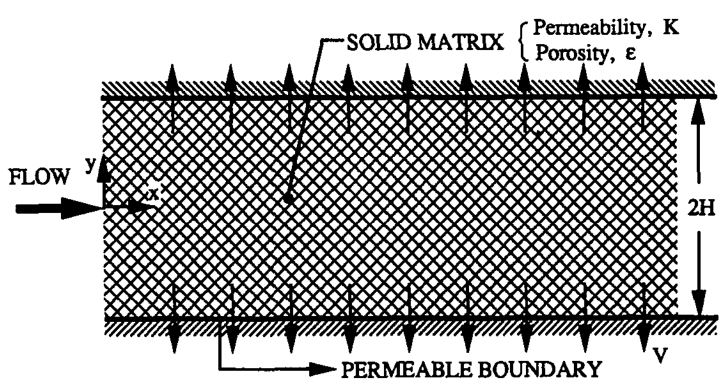

Fig. 1. Schematic diagram of the porous medium channel bounded by two permeable parallel walls.

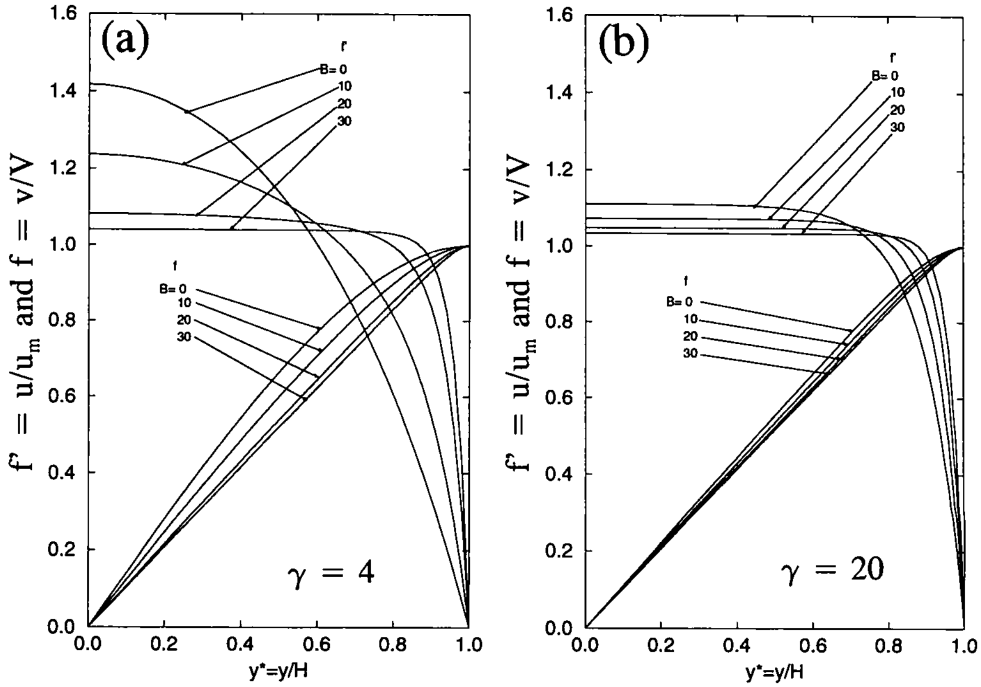

FIG. 2. Variation of the u and v velocity profiles with B for: (a) $\gamma = 4$ and (b) $\gamma = 20$.

Therefore, the Nusselt number in equation (5) is not an arbitrary parameter. In fact, the Nusselt number has to be guessed in order to solve equation (5), such that the solved temperature profile satisfies the condition $\theta'(1) = -Nu$. Equations (4) and (5) combined with the corresponding boundary conditions were solved by using the Runge–Kutta method.

RESULTS AND DISCUSSION

A. *Flow field*

Profiles of both components of the velocity vector are presented in Figs. 2(a, b), for $\gamma = 4$ and 20. The limiting case of $B = 0$ corresponds to the situation of non-permeable walls for which the analytic expression has been reported [1]. For a fixed γ, as the blowing Reynolds number increases, the presence of a flat portion for the axial velocity component near the symmetry plane becomes more pronounced and the thickness of the momentum boundary layer near the wall decreases. For a given B, this behavior becomes more marked as the shape parameter is increased. The velocity component in the y-direction monotonically decreases ($V \to 0$) as one moves from the wall to the mid-plane. For high values of the blowing Reynolds number or the shape parameter, these velocity profiles tend toward linear variations.

The axial velocity always reaches a maximum at the mid-plane. The non-dimensional frictional drag coefficient can be shown to be: $\mathscr{F}\,Re = 4|f''(1)|$, where $Re = u_m(x)H/\nu_f$. The variations of the maximum velocity and the frictional drag coefficient are illustrated in Fig. 3. For a given γ, as

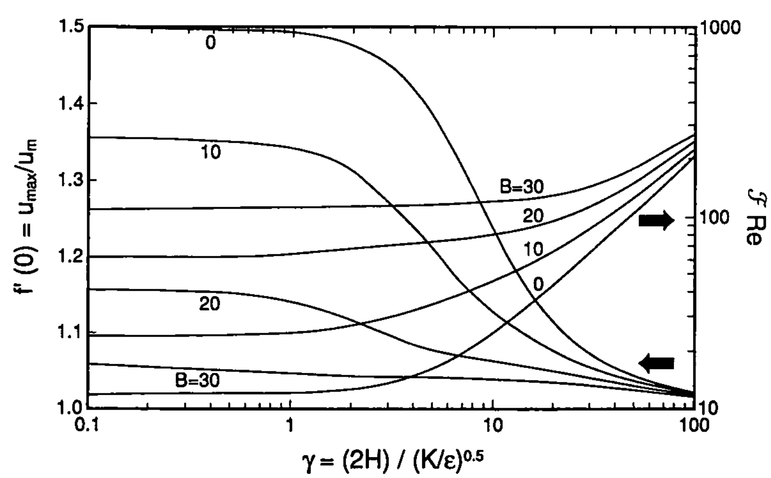

FIG. 3. Dependence of the maximum axial velocity and the frictional drag coefficient on the porous medium shape parameter and the blowing Reynolds number.

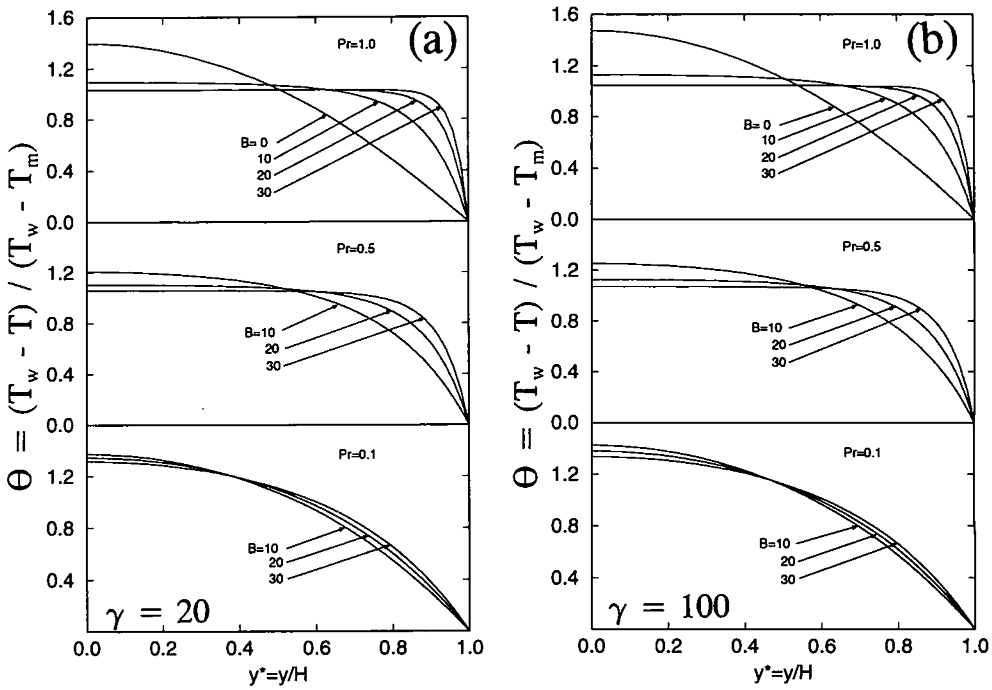

FIG. 4. Variation of the temperature profiles (Pr = 0.1, 0.5 and 1) with B for: (a) γ = 20 and (b) γ = 100.

the blowing Reynolds number is increased, the maximum velocity decreases due to the flattening of the velocity profiles. For very large values of γ, the maximum velocity approaches unity regardless of the blowing Reynolds number. With no permeation at the walls, $\mathscr{F}\,Re$ approaches 12, whereas for very large shape parameters, it approaches 2γ. Finally, for a given γ, as the permeation is strengthened, frictional drag increases as a result of steep shearing.

B. *Temperature field*

The temperature profiles are presented in Figs. 4(a, b), for γ = 20 and 100. Three different Prandtl numbers (Pr = 0.1, 0.5 and 1) were studied in each figure. For the case of $B = 0$, an analytic expression exists, of the form:

$$\theta|_{B=0} = \frac{[2(1-e^{-\gamma})-\gamma-\gamma e^{-\gamma}]\left[e^{0.5\gamma(y^*-1)}+e^{-0.5\gamma(y^*+1)}-(1+e^{-\gamma})\left(\frac{\gamma^2}{8}(y^{*2}-1)+1\right)\right]}{\left[-5(1-e^{-2\gamma})+(1+e^{-\gamma})^2\left(-\frac{\gamma^3}{12}+2\gamma\right)+2\gamma e^{-\gamma}\right]}. \quad (6)$$

In the case of non-porous media, relation (6) becomes $\theta = (35/136)(y^{*2}-1)(y^{*2}-5)$, whereas with $\gamma \to \infty$, it becomes a parabola: $\theta = 1.5(1-y^{*2})$. For a given γ and Pr, as the blowing Reynolds number increases, the temperature profile near the symmetry plane becomes flatter and the temperature gradient at the wall increases markedly, analogous to the axial velocity profiles. As expected, for a given flow condition (fixed γ and B), the above-mentioned features of the temperature profile become more pronounced for the high-Pr fluids. Finally, for a given γ, one can note that the temperature profile is only a function of the Peclet number.

The dependence of the Nusselt number on the porous medium shape parameter and the Peclet number is illustrated in Fig. 5. For the case of $Pe = 0$, an analytic expression exists

$$Nu|_{B=0} = \frac{-[2(1-e^{-\gamma})-\gamma-\gamma e^{-\gamma}]^2}{4\left[-\frac{5}{\gamma}(1-e^{-2\gamma})+(1+e^{-\gamma})^2\left(-\frac{\gamma^2}{12}+2\right)+2e^{-\gamma}\right]}. \quad (7)$$

For non-porous media, relation (7) becomes $Nu = 35/17 \approx 2.0588$, whereas with $\gamma \to \infty$, $Nu \to 3$, indicating heat transfer enhancement of about 50%. For a given γ, permeation at the walls brings about more enhanced heat transfer. For low Pe, the Nusselt number exhibits a dependency on the shape parameter, whereby as γ is increased, heat transfer is augmented. On the other hand, for high Pe, the Nusselt number is independent of the shape parameter. For the high Peclet numbers studied, the Nusselt number is almost equal to the Peclet number.

CONCLUSIONS

For a given porous medium shape parameter, as the blowing Reynolds number increases, profiles of the axial velocity component and temperature become flatter near the symmetry axis and the respective gradients near the wall increase markedly. For a given blowing Reynolds number, this behavior becomes more marked as the shape parameter is increased. The velocity component in the y-direction monotonically decreases as one moves away from the wall and finally vanishes at the symmetry plane. For a given shape parameter, as the permeation at the walls is strengthened,

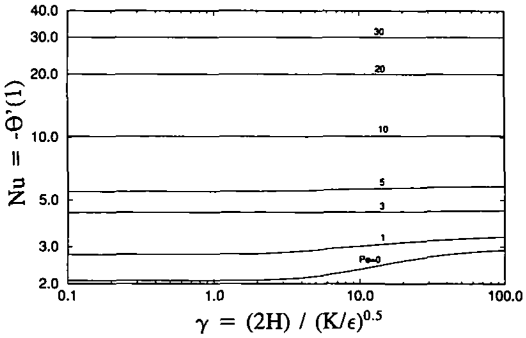

FIG. 5. Dependence of the Nusselt number on the Peclet number and the porous medium shape parameter.

frictional drag increases as a result of steep shearing next to the walls. For low Peclet numbers, the enhanced Nusselt number exhibits a dependency on the shape parameter. For high Peclet numbers, the Nusselt number is independent of the shape parameter.

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